Reply to "Comment on 'Low-frequency character of the Casimir force between metallic films' "

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We respond to the "Comment on 'Low-frequency character of the Casimir force between metallic films,' " by G. Bimonte, which points out an error in our earlier work [Phys. Rev. E **70**, 047102(R) (2004)]. In particular, after correcting the error, the frequency range of the finite temperature contribution to the Casimir force is expanded and exceeds the range of validity for use of the Leontovich boundary. We estimate the size of the thermal effect with a modified boundary condition, and show that it agrees with the result of Bimonte, within the errors of previous experiments. An accurate (subpercent) calculation of the finite temperature contribution remains a theoretical challenge.

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In the Comment on our paper [1], it is pointed out that an error was made in the treatment of the TE electromagnetic mode in that we did not allow for a $\hat{z}$ component of the magnetic field. We agree with most of the analysis presented in the Comment. The results presented in the Comment suggest an 8% *increase* in the finite-conductivity corrected Casimir force, which is within the combined experimental and theoretical error of a previous long-range experimental result [2], particularly at the 1 μm plate separation. Both the experimental and Comment results are in conflict with previous theoretical work that suggests up to 40% *reduction* in the Casimir force [3]. In light of this, it is useful to question all assumptions that have been incorporated in the Comment and other theoretical work, (e.g, [4], where the surface impedance approach was first used), and to assess the accuracy with which the finite temperature correction can be calculated.

It is not obvious from the plot in Fig. 2 of the Comment, but the thermal correction becomes negative for frequencies above 5×10^{10} s^{-1}, so, in fact, the correction depends on where the upper limit of the C_2 integration is cut off. The cutoff seems to be chosen as 10^{13} s^{-1}; extending the upper limit of the integral to where it converges (and outside the range where the form of the surface impedance is valid) showing a correction of a factor of two smaller than in the Comment.

The result presented in Fig. 2 of the Comment, indicating that the evanescent wave contribution to the thermal correction has a significant very low-frequency contribution, is quite intriguing. Unfortunately, at such low frequencies, the Leontovich boundary conditions (Eqs. (1) and (2) of the Comment, or Sec. 67 of [5]) cannot be used. The Leontovich boundary condition is very useful because one does not need to consider the field inside the material. However, this treatment only works when the skin depth is much shorter that the mode wavelength, or alternatively, when the field gradient in the material is much steeper than the normal or parallel field gradients in the vacuum above the plates.

For the evanescent modes, it can be shown numerically that the modes that contribute most to the thermal correction have parallel and perpendicular wave numbers nearly equal with value $K\approx1/2a$ where a is the plate separation, independent of frequency, in the range of interest. On the other hand, major contributions to the thermal correction, as shown in Fig. 2 of the Comment, occur for $\omega<10^8$ s^{-1}. For Au, at this frequency, the skin depth for conductivity $\sigma\approx3\times10^{17}$ s^{-1} is $\delta=c/\sqrt{2\pi\sigma\omega}=2\times10^{-3}$ cm$\gg2a=2\times10^{-4}$ cm. So we see the requirements for the Leontovich boundary conditions are badly violated; in fact, they were barely satisfied in [1].

The question of what wave number to use for a nonpropagating evanescent wave in the conducting material, excited by an evanescent wave (in both the directions parallel and perpendicular to the plates) in the vacuum between the plates, has no obvious answer, but was addressed in [1]. There is no fixed relation between ω and K; both can be, in essence, chosen independently.

It is still possible to derive an effective surface impedance relating H_x and E_y, but clearly Eq. (2) of the Comment cannot apply at low frequencies when $\delta\ll1/K$, or, in the case of the experiment [2] where a thin metallic film of thickness d was used, when $d\ll\delta$ (and $d\ll1/K\approx2a$). In the latter case, the electric field is constant through the film and introduces a current $j_y=\sigma E_y$. From Maxwell's equation $\int_C\vec{B}\cdot d\ell=\oint_S[(4\pi/c)\vec{j}-(i\omega/c)\vec{E}]\cdot\hat{n}d\,S$, and integrating a rectangular small loop in the x-z plane through the film and along the surfaces, implies a relationship between E_x and H_y,

$$H_x=\pm\left[\frac{2\pi\sigma d}{2c}-i\frac{\omega d}{c}\right]E_y\rightarrow\zeta=\left[\frac{2\pi\sigma d}{c}-i\frac{\omega d}{c}\right]^{-1},\quad(1)$$

when $\delta>d$ and $2a>d$. With this modified impedance, the thermal contribution to the Casimir force calculated as in the Comment at $a=10^{-4}$ cm, and for $d=5\times10^{-5}$ cm, is reduced by a factor of 2, but, in fact, depends strongly on the cutoff frequency, the choice of which can change both the sign and magnitude of the net total correction.

In the case of infinitely thick plates, it would probably be reasonable to take $d=1/K\approx2a\ll\delta$ as an effective film thickness, for sufficiently low frequency, in which case ζ in Eq. (1) has to be divided by a factor of 2 to account for the fields being zero deep in the plates. This does not, however, change the qualitative results discussed above.

Because the form of the surface impedance depends on frequency in a more complicated way than Eq. (2) of the

Comment, calculating the finite temperature correction, of order 10% of the total Casimir force at a 1 μm plate separation, to better than 50% accuracy appears to be a daunting task. Unfortunately, without a robust theoretical prediction, there seems to be little point in performing an improved large-plate-separation experiment to address the thermal correction. However, such corrections are important for other fundamental experiments, e.g., searches for non-Newtonian short-range forces, so continued experimental and theoretical studies have ample motivation. We hope that this Response will spur further theoretical work to address the issues that have been brought forward.

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